UNIVERSITE DE LIEGE
Faculté des Sciences Appliquées

AN INTEGRATED APPROACH TO MECHANISM ANALYSIS

Thèse presentée par

Alberto CARDONA

en vue de l'obtention
du grade scientifique
de Docteur en Sciences Appliquées

Année académique 1988-1989

ACKNOWLEDGEMENTS

I wish to acknowledge Professor Michel Géradin, who supervised this research and provided me the most appropriate work environment at the department *Dynamique des Constructions Mécaniques.*

All my thanks are due to the people of the *Laboratoire de Techniques Aéronautiques et Spatiales* for their cooperation to the completion of this work, and for the cordial welcome I received at Liège.

I would like to express my appreciation to Dr. Sergio Idelsohn, who initiated me in research and finite elements at Santa Fe.

I gratefully acknowledge the financial support of the *Consejo Nacional de Investigaciones Científicas y Técnicas de la República Argentina.*

Finally, I would like to express my deep gratitude to Claudia, Juan y Paula, who, through their patience, made a major contribution to this work.

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Introduction

The determination of displacements and efforts in members of spatial mechanisms requires a particular technique of analysis that is not considered in most standard codes of structural analysis. In fact, these ones have been conceived with the aim of analyzing structures fixed or almost fixed in space. Thus, they adopt some simplifying hypotheses of linearization of rotations and inertial terms. These hypotheses are no longer correct, for instance, in the extremely simple case of a crank slider mechanism.

In the past, most mechanisms as vehicles, robots and antennas have been modeled as rigid multibody systems. The rigid-body assumption served as a basis for almost all the initially developed codes for mechanism analysis. This assumption gives good results as long as the elastic deformation in members remain small, the case in many practical mechanical systems.

The performance specifications for newly developed mechanisms usually require high speeds and lightweight construction, while imposing additional criteria on design procedures. Typical requirements are: reduction of power consumption, increase of external loading, reduction of acoustic radiation, and generation of more accurate output characteristics. Elasto-dynamic phenomena that are of little consequence in operating low-speed mechanisms, present a non negligible effect under current working conditions. Links vibrate due to their inherent flexibility and to the more severe force fields that act upon them. The traditional rigid analyses are inadequate for the analysis since, by definition, these models are unable to represent the phenomena one intends to analyze.

Technological applications arise in several disciplines, such as machine design, robotics, aircraft dynamics and spacecraft dynamics. Elastic linkages, rotating machinery, robot manipulator arms, aircraft propellers, helicopter or turbine rotor blades, flexible satellites and earth orbiting large space structures furnish some examples. Another example of interest appears when the kinematic structure of the mechanism involves closed loops, as for instance in some designs of folding antennas. The state of such a mechanism is very sensitive to small changes in geometric parameters: even small deformations of particular bodies may result in large motions of the rest of the system.

In some cases elastic deformations are even desirable. For instance, in some mechanisms conceived to be deployed in space, member flexibility provides the mobility to the system. If members were rigid, the system would become a structure. Another example is furnished by off-road vehicle design, where the relatively soft chassis contributes to good ride behavior.

Mechanism analysis can be seen as a branch of structural dynamics. Several particular aspects characterize this discipline, giving to it a special flavor. The techniques for analysis of mechanisms have progressed steadily over the past twenty years. The field has initially evolved well separated from finite element structural dynamics, but nowadays, both techniques are more and more interrelated. In this thesis, we look at the mechanism as any other case of structural dynamics. We put into evidence its own peculiarities and we discuss the special techniques that should be applied to solve the problem, by always trying to rest within the well-known techniques of analysis already developed in structural dynamics.

Towards the conception of a general purpose program for the analysis of mechanisms

The discussion that follows shows some of the options the analyst is faced to in order to conceive a general purpose mechanism analysis program.

Two principal systems of coordinates were introduced in the literature to deal with the kinematics of rigid multibody systems:

1. The method of Lagrangian coordinates, largely used in robotics [1-3], has the merit of keeping to a minimum the number of generalized coordinates and of giving direct access to the displacements at joints. However, it suffers from such drawbacks as complex description of multi-loop mechanisms, relatively difficult generalization to flexible mechanisms and high degree of nonlinearity.
2. The method of Cartesian coordinates [4-6] has specific attractive features such as generality, lower degree of nonlinearity than the previous approach and easy topological

description. Coupling is simply expressed as a set of constraints at joints. As a result, the number of kinematics unknowns is significantly higher than with the Lagrangian coordinates approach. However, expressing and treating the constraints becomes inefficient from a computational point of view when dealing with flexibility effects in 3-D motion.

Both techniques share the preferences of analysts. The Lagrangian coordinates are usually applied to model small systems with particular topology, while the Cartesian coordinates find application into modeling general systems of any size and with any topology. The finite element method, in the standard form one knows from structural analysis, may be seen as a variant to the Cartesian coordinates approach [7-12]. However, finite elements do not necessarily imply the use of Cartesian coordinates, since many programs make finite elements using Lagrangian coordinates (see, e.g. [3,13]).

Techniques for modeling flexibility effects in mechanisms have naturally evolved from the initially developed rigid body models. The finite element method gave the theoretical basis to develop most existing codes for analysis of flexible mechanisms. Usually, their modeling capabilities are either restricted to bodies composed of beam elements or to bodies of any configuration whose behavior is represented by a modal expansion. Then, two different techniques of formulating the equations of motion can be distinguished:

1. Most flexible analyses assume that the absolute motion of each link may be decomposed into a rigid-body displacement upon which is superposed a small deformation [14]. This deformation is measured relative to a local coordinate system fixed in the element in an undeformed reference state. A *floating frame* approach has also been proposed [11], in which the coordinate system is not strictly fixed to the element but it suffers slight orientation changes with deformation. With the assumption of small strains, the use of a local frame allows a simple expression for the total potential energy of the structure. By contrast, the expression of the kinetic energy of the system takes a rather cumbersome form. The resulting equations of motion, although restricted to small strains, are nonlinear and highly coupled in the inertia terms due to the Coriolis and centrifugal effects, as well as inertia due to rotation of the floating frame.

2. A methodology that represents a full departure from traditional approaches is based on referring the motion of the system to the inertial frame. Then, the kinetic energy of the system is reduced to a simple quadratic uncoupled form, resulting in a drastic simplification of the inertia operator [15-18]. The stiffness operator becomes now nonlinear; the essential step needed in developing this alternative is the use of finite deformation structural theories – rods, plates, shells, three-dimensional continua – whose appropriate strain measures have the required property of invariance with respect to rigid body motions.

Again, both options find their own range of application. The first one continues to be employed, mainly, when modeling large structural assemblies composed by many finite elements, resulting in the superelement technique or component modes or any other technique of second-stage discretization. In this way, mechanism analysis programs can take advantage of the general modeling capabilities of existing linear analysis codes. The only limitation is that the deformation of the link in the local frame should remain small.

Many beam finite elements for mechanisms have been developed by following also this idea. However, a common characteristic of mechanism links is their large flexibility as one tries to reach the limit of resistance of the material. This flexibility often makes inappropriate the assumption of linearized strains, and the beam elements based on the first technique of representation are no longer correct. For this reason, the secondly mentioned method of discretization is almost considered of mandatory application in most newly developed codes.

The representation of finite rotations poses a severe difficulty in the development of mechanism analysis codes. The problem lies in the algebraic structure of rotations. Finite rotations in 2 dimensions can be easily handled since they form a vector space. Then, the composition of two consecutive rotations is simply given by the (trivially commutative) addition of the two angles. Many initially developed programs were limited to the handling of planar mechanisms owing to the simplification in manipulating rotations.

On the other hand, finite rotations in three-dimensional space do not form a vector space. Then, for instance, one should make complex operations to compose successive rotations. A number of essential questions arises as to which is the nature of the underlying differential structure and which is the better technique of representation. Some answers to the first question can be found by appealing to techniques of differential geometry [15] A variety of techniques of representation have been proposed in the literature, trying to arrive to the best solution to the second question. This includes:

1. Euler and Bryant angles have been used traditionally to represent finite rotations in rigid body dynamics, vehicle dynamics and dynamics sciences in general. Ref.[19] employed Euler angles in a program for multibody dynamics. Later, these authors changed the system of representation since they found several inconveniences due to the singularity of Euler angles for certain magnitudes of rotation.

2. Euler parameters are currently employed in most rigid multibody dynamics codes. They form a set of four parameters, linked together by a normality constraint. Quaternion algebra [20-24] puts into evidence a series of helpful properties of this set of parameters and their relation to finite rotations. Among their attractions, we can mention a small degree of nonlinearity and no singularities for any rotation. Their main disadvantage is the redundancy that obliges to add a constraint of normality, leading to five degrees of freedom at each node to represent rotations.

3. Many methods have been proposed to represent rotations using only three parameters,

e.g. Rodrigues parameters, the conformal rotation [25-26] vector and the rotational vector [27]. The first set is discarded since it presents a singularity at certain magnitudes of rotation. Both other sets have been used with success to model dynamic problems. We have employed the rotational vector in this work, for reasons that range from simple geometrical meaning to no redundancy in description.

The formulation of joints constitutes a central aspect in the design of a reliable program for mechanism analysis. The formulation should be flexible enough to allow an easy introduction of the large variety of existing joints with a minimum effort for the analyst. At the same time, it should have a small degree of nonlinearity, so as to not generate a time consuming nucleus in the program. We will see in chapter 3 that this aspects are closely related to the definition of a good formulation for handling rotations. We can distinguish two main approaches in the literature:

1. When using Lagrangian coordinates, joints are easy to formulate since one has direct access to the internal displacements at joints. The main drawback relies on the difficulty to generate the equations of complex mechanisms in a systematic way.

2. Cartesian coordinates allow to generalize and systematize the formulation of joints. The number of degrees of freedom is increased, owing to the presence of Lagrange multipliers. Besides, these ones introduce some problems – for instance, instability of constraints – during the time integration that ought to be specially taken into account.

The approach of Cartesian coordinates was followed since it is the only one that allows to generalize and systematize the formulation for all kinds of mechanism topology – a crucial point for a general purpose program.

The latter aspect to consider is the time integration algorithm. There exist, again, a close inter relation with previously mentioned points, e.g. the formulation of rotations and the formulation of joints. We have already mentioned that the constraints introduce an destabilizing effect into the time integration. Several procedures have been proposed to overcome this problem:

1. Some authors employ the method of coordinate partitioning [28]. The basis for this method is to determine a set of independent coordinates and afterwards integrate in this subspace. The method of singular value decomposition also pursues this objective [29].

2. Many codes use the method of constraint stabilization initially proposed in ref.[30]. It is based on modifying constraints by adding some terms of stabilization. It has the drawbacks of never exactly verifying the constraints, and of depending on a set of rather arbitrarily determined constants in the formulation. Ref.[31] proposed a method to stabilize constraints based on a *staggered stabilized procedure*, allowing to formulate a two-stage staggered explicit algorithm.

3. A radically different approach is to stabilize constraints be means of numerical dissipation in the time integration algorithm. In this method, the dissipation properties of certain integration algorithms are employed to stabilize the constraints. The approach has the advantage of being consistent with the usual approaches in structural dynamics, without any dependency on arbitrary constants and exactly satisfying the constraints of the formulation.

There exists another factor to be considered in the formulation of a time integration algorithm for multibody dynamics. Finite rotations do not form a vector space, but standard time integrators are designed to work with values in a vector space. Then, a particular treatment ought to be made to the rotation associated terms in order to compute the solution, which is not considered in standard structural dynamics programs. The idea is to project the equations of motion onto the tangent space at the consider rotation, so as to obtain equations which now take values on a vector space.

Outline of this report

The general outline of the report follows. In chapter 1, we make a general presentation of the problem of representing finite rotations in three dimensional space. The different existing techniques of representation are developed. We establish some criteria to follow in order to select a suitable technique of representation, to be used in a general purpose program for analysis of mechanisms. We give also relations to calculate angular velocities and accelerations in all parameterization systems. We conclude by selecting the technique of representation to be used elsewhere in this work.

Chapter 2 begins with a discussion about the equations of rigid body dynamics in terms of the selected system for parameterizing rotations. We develop then a nonlinear theory for a beam and a Reissner-Mindlin beam element. The element is capable of suffering large displacements and rotations, but deformations remain within the elastic range. Inertia terms are introduced by marking the analogy existing between the rigid body equations and the equations of the plane section. A kind of updated Lagrangian formulation for the rotational degrees of freedom is described, that allows to handle rotations of arbitrarily large magnitudes in three dimensional space using the rotational vector technique.

In chapter 3 we introduce joints. After making a general description of the procedures used to impose constraints into a dynamic system, we set the basis for a general procedure to follow in order to describe the equations governing joints. Joints of both holonomic and non-holonomic type are considered. They include the lower-pair joints – hinge, cylindrical, prismatic, etc. – as well as some higher-pair joints: slider, Hooke, wheel, etc. In all cases, an augmented Lagrangian procedure was employed to generate the equations of motion.

In chapter 4 we discuss an implementation of the component-mode method for multibody dynamics. In it, flexible bodies are represented by a collection of fixed-boundary modes plus some constraint modes. The approach is fully nonlinear, allowing to consider large relative rotations between bodies. The only limitation is that the deformation inside the body should be small enough to consider that the elastic behavior of the body remains linear in a local frame. The approach has the advantage of allowing to represent bodies by using the large variety of finite elements existing in the library of a general purpose program for linear dynamic analysis.

In chapter 5, we analyze the equations of motion of an active member in a mechanism. In particular, we treat the case of a hydraulic actuator. The actuator is modeled by using the finite element modular concept. We develop an element, which is integrated into the mechanism analysis package. It fully interacts with all elements, allowing to model complex interaction phenomena between the structure and the hydrodynamics of the actuator itself: for instance, pressure oscillations and peaks inside the chambers of the actuator are correctly predicted.

Chapter 6, finally, presents the way we integrate the equations of motion. We begin by making a general discussion about the stability of constrained dynamic systems. We show that the Newmark's algorithm does not give correct solutions for this kind of systems, since it contains a weak instability which is excited for all values of the time step. We consider then the application of dissipative algorithms, like for instance, the Hilber-Hughes-Taylor one. We finish this chapter by discussing the implementation of algorithms for integrating the equations of motion in the presence of large finite rotations.

Examples illustrating the different findings of the theory are presented at the end of chapters 2 to 6. The examples are organized so as to show the implications of the theory of each chapter. At the same time, the reader can evaluate the power of the mechanism analysis package **MECANO** developed as an application of the concepts presented in this thesis, with some examples taken from real industrial problems alternating with purely academic ones.

A recapitulative about the original features of this thesis is presented in the *Conclusion*.

References

1. WITTENBURG J., *Dynamics of systems of rigid bodies*, Teubner, Stuttgart (1977).

2. PAUL R., *Robot manipulators. Mathematics, programming and control*, MIT Press, Cambridge, Mass.(1981).

3. GERADIN M., ROBERT G. and BERNARDIN, "Dynamic modelling of manipulators with flexible members", in *Advanced software in robotics*, Ed. by A. DANTHINE and M. GERADIN, North Holland (1984).

4. CHACE M., *Computer-aided engineering of large displacements dynamic mechanical systems*, CAE/CAD/CAM Conference, April 1983, Salt Lake City, Utah.

5. HAUG E. J., WEHAGE R. and BARMAN N., *Dynamic analysis and design of constrained mechanical systems*, Technical Report Nx 50, Div.of Eng., Univ.of Iowa, Nov. 1978.

6. KANE T.R. and LEVINSON D.A., "Multibody dynamics", *Journal of Applied Mechanics*, Vol.50, pp. 1071-1078 (1983).

7. VAN DER WERFF L., *Kinematic and dynamic analysis of mechanisms. A finite element approach*, PhD Thesis, Delft Univ.Press (1977).

8. SONG J. and HAUG E. J., "Dynamic analysis of mechanisms", *Comp. Meth. in Appl. Mech and Engng.* Vol 24, pp.359-381 (1980).

9. VAN DER WERFF K. and JONKER J. B., "Dynamics of flexible mechanisms", in *Computer aided analysis and optimization of mechanical system dynamics*, Ed.E. J. HAUG, Springer-Verlag (1984).

10. SHABANA A. A., "Substructure synthesis methods for dynamic analysis of multibody systems", *Computers and Structures*, Vol.20, pp.737-744 (1985).

11. AGRAWAL O. P. and SHABANA A. A., "Application of deformable-body mean axis to flexible multibody system dynamics", *Comp. Meth. Appl. Mech. Engng.*, Vol.56, pp.217-245 (1986).

12. NIKRAVESH P., "Spatial kinematics and dynamic analysis with Euler parameters",

in *Computer aided analysis and optimization of mechanical system dynamics*, Ed. by E. J. HAUG, Springer-Verlag (1984).

13. SINGH R.P., VANDER VOORT R.J. and LIKINS P.W., "Dynamics of flexible bodies in tree topology – A computer-oriented approach", *J. Guidance*, Vol. 8, pp. 584-590 (1985).

14. THOMPSON B.S. and SUNG C.K., "A survey of finite element techniques for mechanism design", *Mechanism and Machine Theory*, Vol. 21 , pp. 351-359 (1986).

15. VU-QUOC L., *Dynamics of flexible structures performing large overall motions: a geometrically-nonlinear approach*, Mem. UCB/ERL M86/36, Univ. of California, Berkeley (1986).

16. KONDOH K., TANAKA K. and ATLURI S. N., "An explicit expression for the tangent-stiffness of a finitely deformed 3-D beam and its use in the analysis of space frames", *Computers and Structures*, Vol.24, pp.253-271 (1986).

17. SIMO J. C. and VU-QUOC L., "A three-dimensional finite strain rod model. Part II: computational aspects", *Comp. Meth. Appl. Mech. Engng.*, Vol.58, pp.79-116 (1986).

18. PARK K.C., *Flexible beam dynamics: part I - formulation*, Center for Space Structures and Control, University of Colorado, Boulder (1986).

19. CHUNG I., CHANG C., HAUG E.J. and WEHAGE R.A., *Dynamic analysis of three dimensional constrained mechanical systems using Euler parameters*, Center for Computer Aided Design, University of Iowa, Report No. 81-11 (1981).

20. WEHAGE R. A., "Quaternions and Euler parameters. A brief exposition", in *Computer aided analysis and optimization of mechanical system dynamics*, Ed. by E. J. HAUG, Springer-Verlag (1984).

21. SPRING K. W., "Euler parameters and the use of quaternion algebra in the manipulation of finite rotations: a review", *Mechanism and Machine Theory*, Vol.21, pp.365-373 (1986).

22. HAMILTON W.R., *Elements of quaternions*, Cambridge University Press, Cambridge (1899).

23. NIKRAVESH P.E., "Spatial kinematic and dynamic analysis with Euler parameters", in *Computer aided analysis and optimization of mechanical system dynamics*, Ed.E. J. HAUG, Springer-Verlag (1984).

24. GERADIN M., "Finite element approach to kinematic and dynamic analysis of mechanisms using Euler parameters", in *Numerical methods for nonlinear problems II*, Ed. by C.TAYLOR et al, Pineridge Press, Swansea, U.K. (1984).

25. MILENKOVIC V., "Coordinates suitable for angular motion synthesis in robots", Proceedings of ROBOT 6 conference (1982).

26. GERADIN M. and CARDONA A., "Kinematics and dynamics of rigid and flexible mechanisms using finite elements and quaternion algebra", to appear in *Computational Mechanics* (1988).

27. ARGYRIS J., "An excursion into large rotations", *Comp. Meth. Appl. Mech. Engng.*, Vol.32, pp.85-155 (1982).

28. WEHAGE R.A. and HAUG E.J., "Generalized coordinate partitioning for reduction dimension in analysis of constrained dynamic systems", *Journal of Mechanical Design*, Vol. 104, pp. 247-255 (1982).

29. SINGH R.P. and LIKINS P.W., "Singular value decomposition for constrained dynamical systems", *Journal of Applied Mechanics*, Vol. 52, pp. 943-948 (1985).

30. BAUMGARTE J., "Stabilization of constraints and integrals of motion in dynamical systems", *Comp. Meth. Appl. Mech. Engng*, Vol.1, pp.1-16 (1972).

31. PARK K.C. and CHIOU J.C., "Evaluation of constraint stabilization procedures for multibody dynamics systems", *Proc. of the* 28^{th} *Structural Dynamics and Materials Conference*, AIAA Paper No.87-0927, pp. 769-773, (1987).