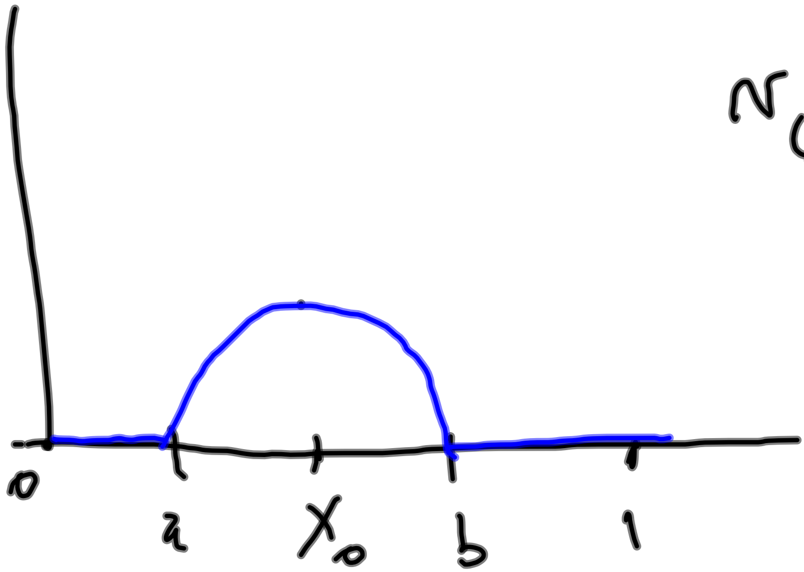
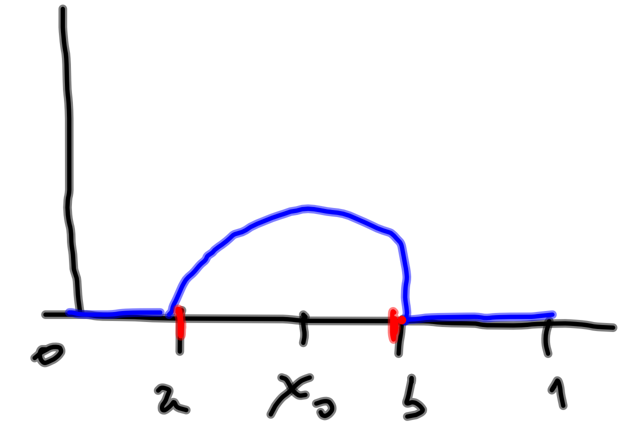



$$v_{(x)} \begin{cases} (a-x)(x-b) & x \in [a,b] \\ 0 & x \notin [a,b] \end{cases}$$

$$[a,b] \in [0,1] \qquad w_{(x)} > 0$$

$$\int_0^1 w_{(x)} v_{(x)} \, dx = 0$$




$$\left.\begin{aligned}&\int w v\,dx = 0\\ &w = 0\end{aligned}\right\}\forall v \in V$$

$$\int_0^1 w v\,dx = \underbrace{\cancel{\int_0^a w v\,dx}}_{v=0} + \int_a^b w v\,dx + \underbrace{\cancel{\int_b^1 w v\,dx}}_{v=0}$$

$$\int_0^1 w v\,dx = \int_a^b w \cdot v\,dx > 0$$

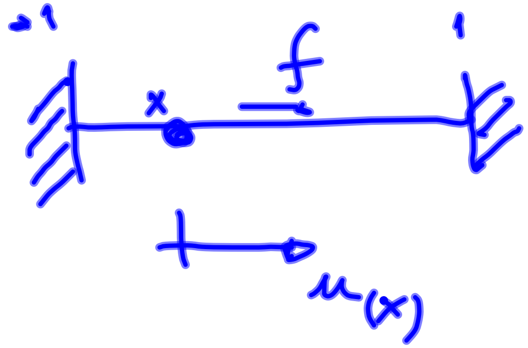
Contradice $\int w v\,dx = 0$

$w \equiv 0 \quad [a, b]$

? Ejercicio

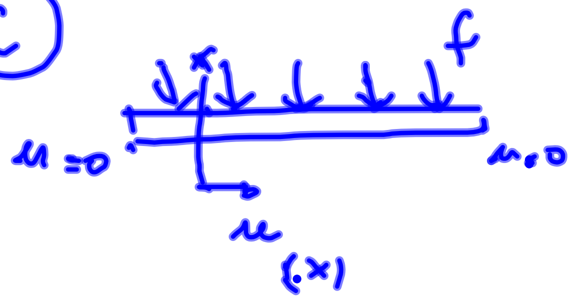
$$\begin{cases} -u''(x) = f(x) & \text{ED} \quad -1 < x < 1 \\ u(-1) = u(1) = 0 & \text{BC} \end{cases}$$

Ⓐ




- Ley Hooke
- Ec. Equilibrio
- Cond. Borde.

Ⓒ

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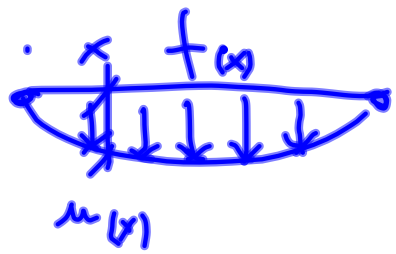


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# Ej. 4

$$(D)\begin{cases} -u''(x) = f(x) & 0<x<1 \\ u(0) = u(1) = 0 \end{cases}$$

1_ Pesamos $v \in V_h$

$v(0) = v(1) = 0$


$$-\int_0^1 u'' \cdot v = \int_0^1 f \cdot v$$

$$-u' \cdot v \Big|_0^1 + \int_0^1 u' v' \, dx = \int_0 f \cdot v \, dx \quad *$$

$$\cancel{-u' v\Big|_0^1} + \int_0^1 u' v' \, dx = \int_0^1 f \cdot v \, dx$$

$$\int_0^1 u' v' \, dx = \int_0^1 f \cdot v \, dx$$

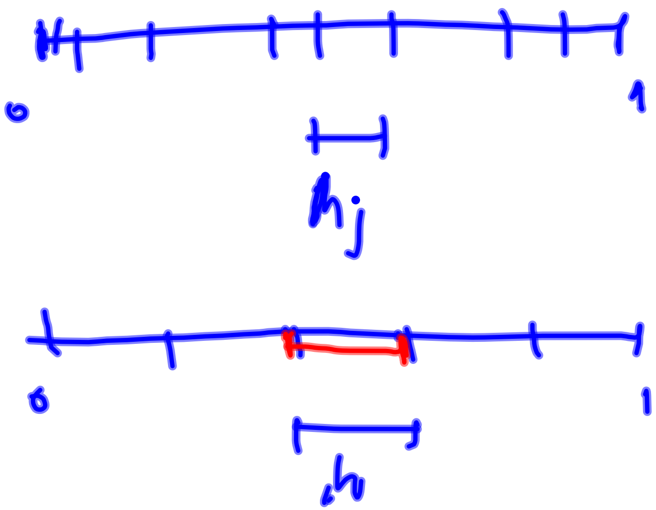
$$\boxed{(u', v') = (f, v)}$$

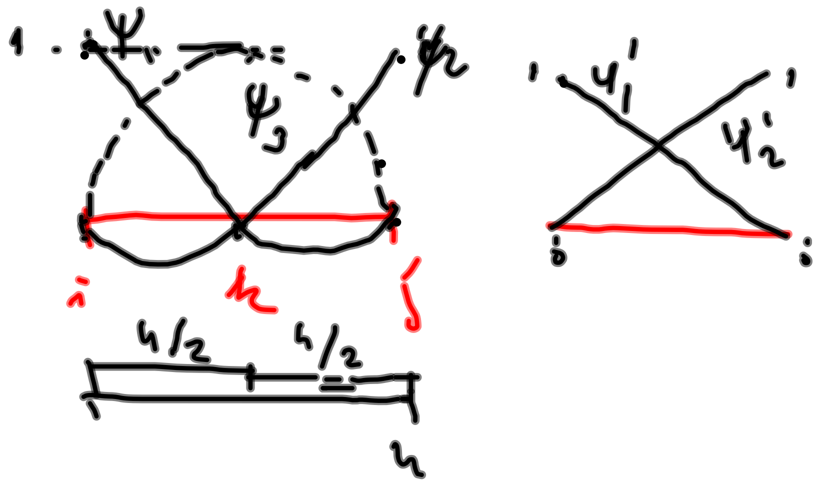
$V_h \{ v \in V :$ cuadráticas a trozos y continua $I = [0,1] \}$

- Prob. Variacional (V)

Hallar $u_h \in V_h \,/\, (u_h', v') = (f, v)$

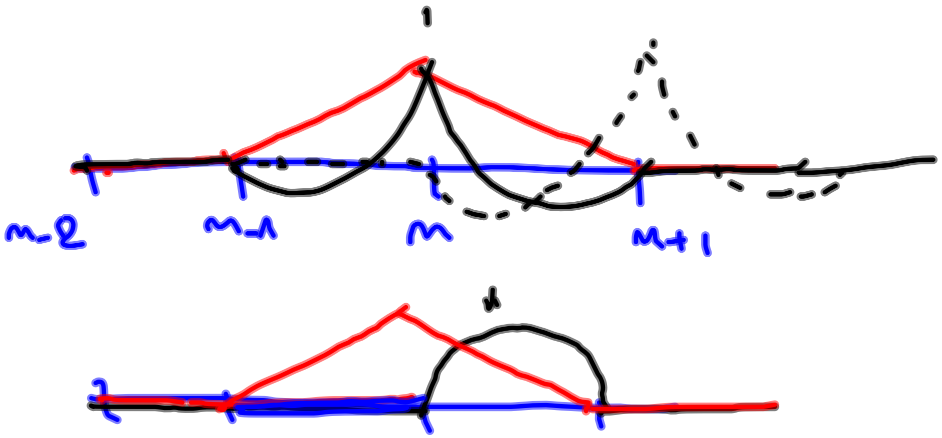
$\forall v \in V_h$

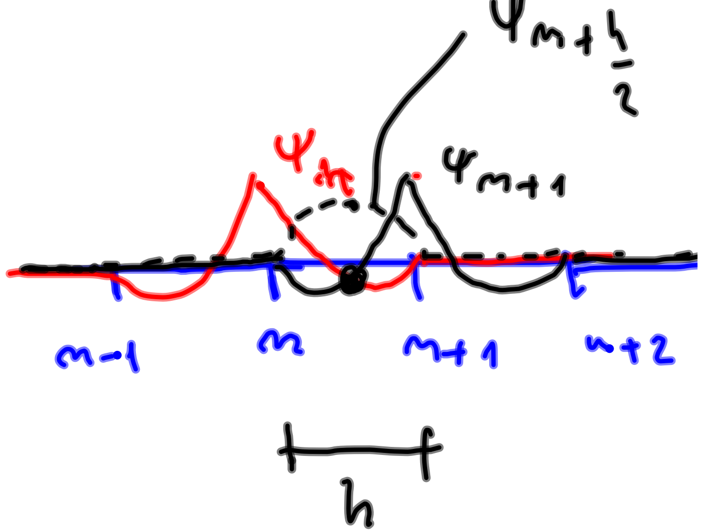


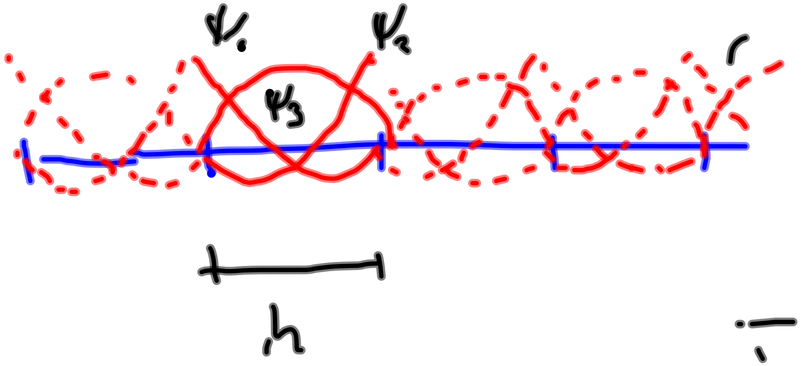



$$\Psi_m \begin{cases} \psi_m & x_{m-1} < x < x_{m+1} \\ \psi_{m+1} & x_m < x < x_{m+2} \\ 0 & \text{en el resto} \end{cases}$$

$$\Psi_m^* \begin{cases} \psi_{m+\frac{h}{2}} & x_m < x < x_{m+1} \\ 0 & \text{en el resto} \end{cases}$$





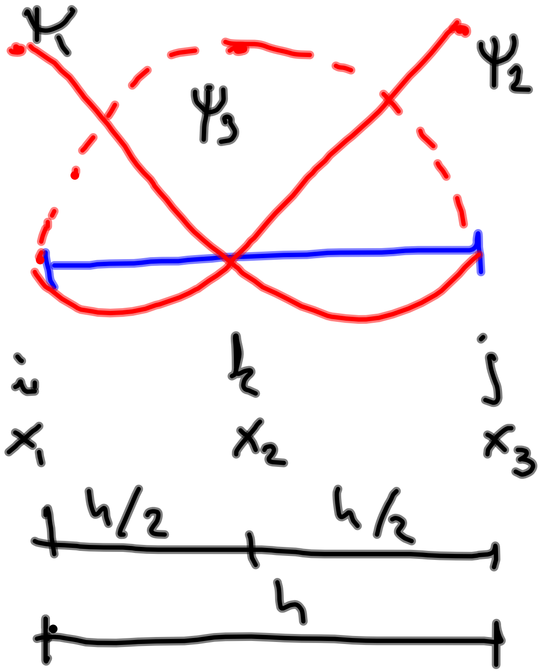

– Polinomio Legendre

$$\psi_1 = \frac{(x - x_2)(x - x_3)}{(x_1 - x_2)(x_1 - x_3)}$$

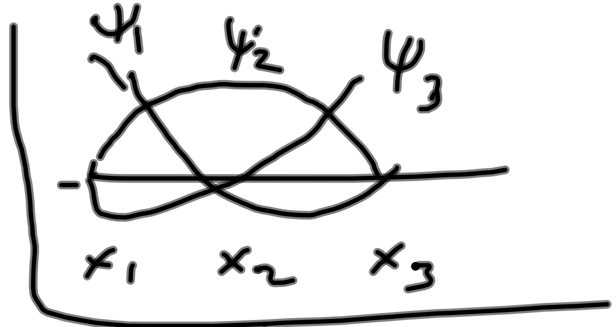
$$\boxed{\psi_1 = \frac{(x - h/2)(x - h)}{(0 - h/2)(0 - h)}}$$

$$\psi_1 = \frac{2}{h^2}\left(x - \frac{h}{2}\right)(x - h)$$

$$\psi_1' = \frac{2}{h^2}\left(2x - \frac{3}{2}h\right)$$



$$\psi_2 = \frac{(x - x_1)(x - x_3)}{(x_2 - x_1)(x_2 - x_3)}$$




$$\psi_2 = \frac{(x - 0)(x - h)}{\left(\frac{h}{2} - 0\right)\left(\frac{h}{2} - h\right)} \longrightarrow \psi_2 = -\frac{4}{h^2}(x)(x - h)$$

$$\psi_2' = -\frac{4}{h^2}(2x - h)$$

$$\psi_3 = \frac{(x - x_1)(x - x_2)}{(x_3 - x_1)(x_3 - x_2)}$$

$$\psi_3 = \frac{(x - 0)(x - h/2)}{(h - 0)(h - h/2)} \longrightarrow \psi_3 = \frac{2}{h^2}(x)\left(x - \frac{h}{2}\right)$$

$$\psi_3' = \frac{2}{h^2}\left(2x - \frac{h}{2}\right)$$

$$A_K = \begin{bmatrix} (\psi_1', \psi_1') & (\psi_1', \psi_2') & (\psi_1', \psi_3') \\ & (\psi_2', \psi_2') & (\psi_2', \psi_3') \\ \text{Sim.} & & (\psi_3', \psi_3') \end{bmatrix}$$

$$(u', v') = (f, v) \qquad \forall v \in V_h$$

$$u_h = \sum_i \xi_i \cdot \psi_i \qquad \xi_i = u_h(x_i)$$

$$v = \psi_j$$

$$\left(\sum \xi_i \psi_i', \psi_j'\right) = (f, \psi_j)$$

$$\left(\sum \xi_i \psi_i', \psi_j'\right) = (f, \psi_j)$$

$$\sum \xi_i \underbrace{(\psi_i', \psi_j')} = \underbrace{(f, \psi_j)}$$

$$\xi \cdot A \quad = b$$

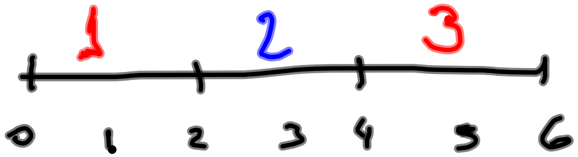
$$A_K = \begin{bmatrix} (\psi_1', \psi_1') & (\psi_1', \psi_2') & (\psi_1', \psi_3') \\ & (\psi_2', \psi_2') & (\psi_2', \psi_3') \\ \text{Sim} & & (\psi_3', \psi_3') \end{bmatrix}$$

en el

$I[i, j]$

$$A_{K_n} \cdot \xi_n = b_n$$

$$\frac{1}{3h}\begin{bmatrix} 7 & -8 & 1 \\ -8 & 16 & -8 \\ 1 & -8 & 7 \end{bmatrix}\begin{bmatrix} \xi_0 \\ \xi_1 \\ \xi_2 \end{bmatrix} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} \qquad \text{local}$$




$$u_0 = u_6 = 0$$

$$\frac{1}{3h}\begin{bmatrix} 7 & -8 & 1 & 0 & 0 & 0 & 0 \\ -8 & 16 & -8 & 0 & 0 & 0 & 0 \\ 1 & -8 & 7+7 & -8 & 1 & 0 & 0 \\ 0 & 0 & -8 & 16 & -8 & 0 & 0 \\ 0 & 0 & 1 & -8 & 7+7 & -8 & 1 \\ 0 & 0 & 0 & 0 & -8 & 16 & -8 \\ 0 & 0 & 0 & 0 & 1 & -8 & 7 \end{bmatrix}\begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{bmatrix}$$

$$
\begin{bmatrix}
16 & -8 & & & \\
-8 & \underline{14} & -8 & 1 & \\
 & -8 & 16 & -8 & \\
 & 1 & -8 & \underline{14} & -8 \\
 & & & -8 & 16
\end{bmatrix}
\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix}
=
\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{bmatrix}
$$

$$
b_k \begin{bmatrix} (f, \psi_1) \\ (f, \psi_2) \\ (f, \psi_3) \end{bmatrix}
$$

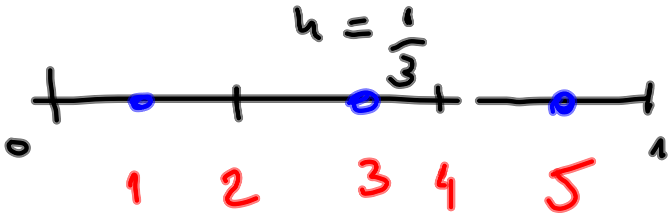
$$f = 1$$

$$b_1 = (1, \psi_1) = \int_0^h \left( \frac{2}{h^2} \left( 2x - \frac{3}{2} h \right) \right) dx$$

$$b_1 = -\frac{h}{6}$$

$$b_2 = (1, \psi_2) = \int_0^h \left( -\frac{4}{h^2} ( 2x - h ) \right) dx$$

$$b_2 = \frac{2}{3} h \qquad b_3 = \frac{h}{6}$$

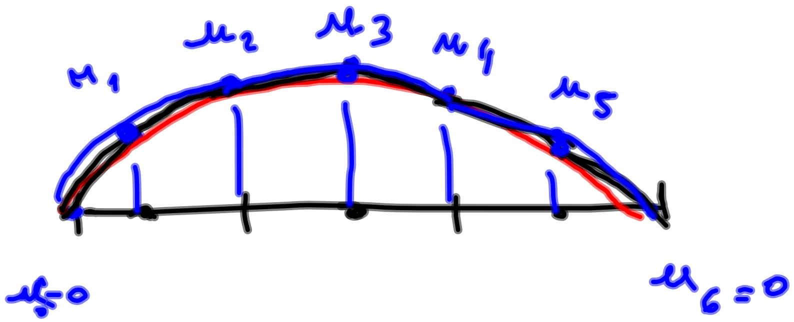



$$b_1 = \frac{1}{18}$$

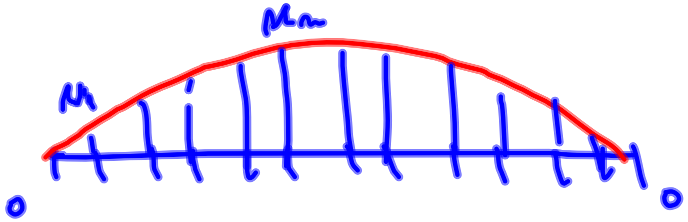
$$b_2 = \frac{2}{9}$$

$$b_3 = \frac{1}{18}$$

$$\begin{bmatrix} 16 & -8 & 0 & 0 & 0 \\ -8 & 14 & -8 & 1 & 0 \\ 0 & -8 & 16 & -8 & 0 \\ 0 & 1 & -8 & 14 & -8 \\ 0 & 0 & 0 & -8 & 16 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} = \begin{bmatrix} 2/9 \\ 0 \\ 2/9 \\ 0 \\ 2/9 \end{bmatrix}$$








ED

$\downarrow$

P (V)

$\downarrow$

$A \cdot \xi = b$

$\xi$