

Resumen de ecuaciones Guía 6 - Ejercicios 1, 2 y 3

| Coordenadas | Campo Velocidad | | Vorticidad ($\Omega_{ij} = -\Omega_{ji}$) | Tasa de deformación ($V_{ij} = V_{ji}$) | Gráficos | Divergencia $\nabla \cdot \mathbf{v}$ |
|---|--|---|--|--|---|--|
| | Func. Potencial | Func. de Corriente | | | | |
| Cartesianas $\Phi(x, y)$ $u(x, y); v(x, y)$ | $u = \frac{\partial \Phi}{\partial x}$ $v = \frac{\partial \Phi}{\partial y}$ | $u = \frac{\partial \Psi}{\partial y}$ $v = -\frac{\partial \Psi}{\partial x}$ | $\Omega_{ij} = \frac{1}{2} \left(\frac{\partial v_j}{\partial x_i} - \frac{\partial v_i}{\partial x_j} \right)$ 2D: $\Omega_{12} = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$ | $V_{ij} = \frac{1}{2} \left(\frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j} \right)$ 2D: $V_{11} = \frac{\partial u}{\partial x}; V_{22} = \frac{\partial v}{\partial y};$ $V_{12} = \frac{1}{2} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)$ | Func. Potencial, Func. de Corriente y Vect. Velocidad | $\nabla \cdot \mathbf{v} = \frac{\partial v_i}{\partial x_i}$ |
| Cilíndricas $\Phi(r, \theta)$ $u_r(r, \theta); u_\theta(r, \theta)$ | $u_r = \frac{\partial \Phi}{\partial r}$ $u_\theta = \frac{1}{r} \frac{\partial \Phi}{\partial \theta}$ | $u_r = \frac{1}{r} \frac{\partial \Psi}{\partial \theta}$ $u_\theta = -\frac{\partial \Psi}{\partial r}$ | 2D: $\Omega_{r\theta} = \frac{1}{2} \left(\frac{\partial u_\theta}{\partial r} - \frac{1}{r} \frac{\partial u_r}{\partial \theta} + \frac{u_\theta}{r} \right)$ | 2D: $V_{rr} = \frac{\partial u_r}{\partial r};$ $V_{\theta\theta} = \frac{u_r}{r} + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta};$ $V_{r\theta} = \frac{1}{2} \left(\frac{\partial u_\theta}{\partial r} + \frac{1}{r} \frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r} \right)$ | Idem. Rotar u_r y u_θ a u y v . $u = u_r \cos \theta - u_\theta \sin \theta$ $v = u_r \sin \theta + u_\theta \cos \theta$ | 2D: $\nabla \cdot \mathbf{v} = \frac{\partial u_r}{\partial r} + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}$ |

| | Vorticidad | | Divergencia $\nabla \cdot \mathbf{v}$ | |
|-------------------|---|---|---|--|
| | Coordenadas Cartesianas | Coordenadas Cilíndricas | Coordenadas Cartesianas | Coordenadas Cilíndricas |
| Función Potencial | 2D: $\Omega_{12} = \frac{1}{2} \left(\frac{\partial^2 \Phi}{\partial x \partial y} - \frac{\partial^2 \Phi}{\partial y \partial x} \right) = 0$ | 2D: $\Omega_{r\theta} = \frac{1}{2} \left(-\frac{1}{r^2} \frac{\partial \Phi}{\partial \theta} + \frac{1}{r} \frac{\partial^2 \Phi}{\partial r \partial \theta} - \frac{1}{r} \frac{\partial^2 \Phi}{\partial r \partial \theta} + \frac{1}{r^2} \frac{\partial \Phi}{\partial \theta} \right) = 0$ | 2D: $\nabla \cdot \mathbf{v} = \frac{1}{2} \left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} \right)$ | 2D: $\nabla \cdot \mathbf{v} = \frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} + \frac{1}{r} \frac{\partial \Phi}{\partial r}$ |
| Función Corriente | 2D: $\Omega_{12} = -\frac{1}{2} \left(\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} \right)$ | 2D: $\Omega_{r\theta} = -\frac{1}{2} \left(\frac{\partial^2 \Psi}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 \Psi}{\partial \theta^2} + \frac{1}{r} \frac{\partial \Psi}{\partial r} \right)$ | 2D: $\nabla \cdot \mathbf{v} = \frac{1}{2} \left(\frac{\partial^2 \Psi}{\partial x \partial y} - \frac{\partial^2 \Psi}{\partial y \partial x} \right) = 0$ | 2D: $\nabla \cdot \mathbf{v} = \frac{1}{r} \frac{\partial \Psi}{\partial r \partial \theta} - \frac{1}{r} \frac{\partial \Psi}{\partial \theta \partial r} = 0$ |