1)

$$\left.\begin{aligned} x_1 &= a_1 \\ x_2 &= \frac{e^{t}(a_2+a_3)}{2} + \frac{e^{-t}(a_2-a_3)}{2} \\ x_3 &= \frac{e^{t}(a_2+a_3)}{2} - \frac{e^{-t}(a_2-a_3)}{2} \end{aligned}\right] \quad (1)$$

$$\left.\begin{aligned} v_1 &= \frac{dx_1}{dt} = 0 \\ v_2 &= \frac{dx_2}{dt} = \frac{e^{t}(a_2+a_3)}{2} - \frac{e^{-t}(a_2-a_3)}{2} \\ v_3 &= \frac{dx_3}{dt} = \frac{e^{t}(a_2+a_3)}{2} + \frac{e^{-t}(a_2-a_3)}{2} \end{aligned}\right] \quad (2)$$

$((b) \to v_i(\underline{a},t))$

Notar:

$$\left.\begin{aligned} v_2 &= x_3 \\ v_3 &= x_2 \\ v_1 &= 0 \end{aligned}\right] \quad ((a) \longrightarrow v_i(\underline{x},t)) \quad (3)$$

También (más largo, proc estándar):
De (1), sumando y restando:

$$a_1 = x_1$$
$$a_2 = \left[e^{-t}(x_2+x_3) + e^{t}(x_2-x_3)\right]/2$$
$$a_3 = \left[e^{-t}(x_2+x_3) - e^{t}(x_2-x_3)\right]/2$$

Reemplazando en (2), se logra el resultado (3).

2) $\phi(x,y) = ax^3 + bx^2y + cxy^2 + dy^3$

$$\sigma_{xx} = \frac{\partial^2\phi}{\partial y^2} = 2cx + 6dy$$

$$\sigma_{yy} = \frac{\partial^2\phi}{\partial x^2} + V = 6ax + 2by + \rho_h g y$$

$$\sigma_{xy} = -\frac{\partial^2\phi}{\partial x \partial y} = -2bx - 2cy = \sigma_{yx}$$

Equilíbrio:

$$\sigma_{ij,j} + X_i = 0$$

$$\underline{X} = \begin{pmatrix} 0 \\ -\rho_h g \end{pmatrix}$$ (cargas de volume)

$$\sigma_{xx,x} + \sigma_{xy,y} + X_x = 2c - 2c + 0 = 0$$

$$\sigma_{yx,x} + \sigma_{yy,y} + X_y = -2b + 2b + \rho_h g - \rho_h g = 0$$

(a)

(1) Eq forç horiz

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -\sigma_{xy} \\ -\sigma_{yy} \end{bmatrix}$$

$$F_{x_{Agua}} = \rho_w g h \times \frac{h}{2}$$

$$\cancel{\int_0^l -\sigma_{xy}\, dx = \int_0^l +2bx\, dx = +bx^2\Big|_0^l = +bl^2}$$

$$\cancel{\therefore\ b = \frac{\rho_w g}{2}\left(\frac{h}{l}\right)^2}$$

$$\sigma_{xy}(x,-h) = 2ch - 2bx$$

$$\int_0^l (-\sigma_{xy})\, dx = -2ch\,x + b\,x^2 \Big|_0^l = bl^2 - 2clh$$

$$\therefore \quad l^2 b - 2lh\,c = \frac{\rho_w g}{2} h^2 \quad (1)$$

II) Eq fuerz vert:

$$F_v = -\frac{\rho_h g l h}{2} \Longrightarrow R_v = \frac{\rho_h g l h}{2}$$ (reacción vertical sobre la presa)

$$\sigma_{yy}(x,-h) = 6ax - 2bh - \rho_h g h$$

$$\int_0^l (-\sigma_{yy})\, dx = 2bh\,x + \rho_h g h\,x - 3a\,x^2 \Big|_0^l =$$

$$= 2bhl + \rho_h g h l - 3a l^2$$

$$\therefore \quad -3l^{\cancel{2}} a + 2h\cancel{l}\, b = -\frac{\rho_h g h \cancel{l}}{2} \quad (2)$$

III) $\sigma_{xx}\big|_A$ = pres hidrostática

$$p = -\rho_w g h \Longrightarrow \sigma = -p = \rho_w g h$$ (Acción sobre la presa)

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix} \begin{Bmatrix} -1 \\ 0 \end{Bmatrix} = \begin{pmatrix} -\sigma_{xx} \\ -\sigma_{xy} \end{pmatrix}$$

$$-\sigma_{xx}\big|_A = -6d(-h) = 6dh$$

$$6d\cancel{h} = \rho_w g \cancel{h} \Longrightarrow d = \frac{\rho_w g}{6}$$

IV) $\sigma_{yy}\big|_B = 0$

$$\sigma_{yy}(l,-h) = 6al - 2bh - \rho_h g h = 0$$

$$6l\,a - 2h\,b = \rho_h g h \qquad (3)$$

De (2) $\longrightarrow$ $$-3l\,a + 2h\,b = -\frac{\rho_h g h}{2}$$

Sumando $$3l\,a = \frac{\rho_h g h}{2}$$

$$a = \frac{\rho_h g h}{6l}$$

$$b = \left[-\frac{\rho_h g h}{2} + 3la\right]\frac{1}{2h} =$$

$$= \left[-\frac{\rho_h g h}{2} + \frac{\rho_h g h}{2}\right]\frac{1}{2h} = 0$$

En(1) $$c = -\frac{\rho_w g \cancel{h^2}}{2\times 2\cancel{h}l} = -\frac{\rho_w g h}{4l}$$

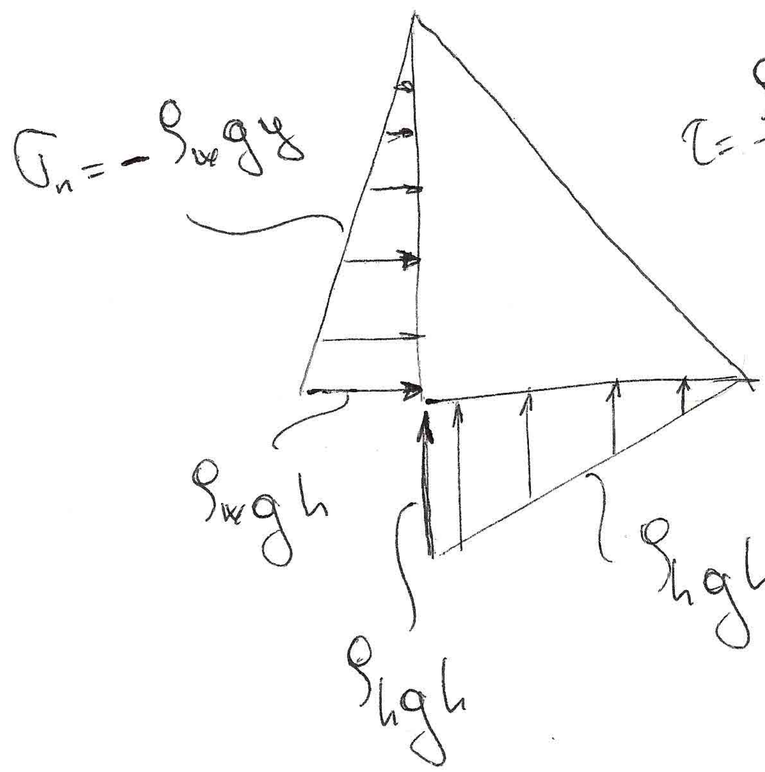
$$\boxed{\begin{aligned} \sigma_{xx} &= -\frac{\rho_w g h}{2l}x + \rho_w g y \\ \sigma_{yy} &= \frac{\rho_h g h}{l}x + \rho_h g y \\ \sigma_{xy} &= \frac{\rho_w g h}{2l}y \end{aligned}}$$

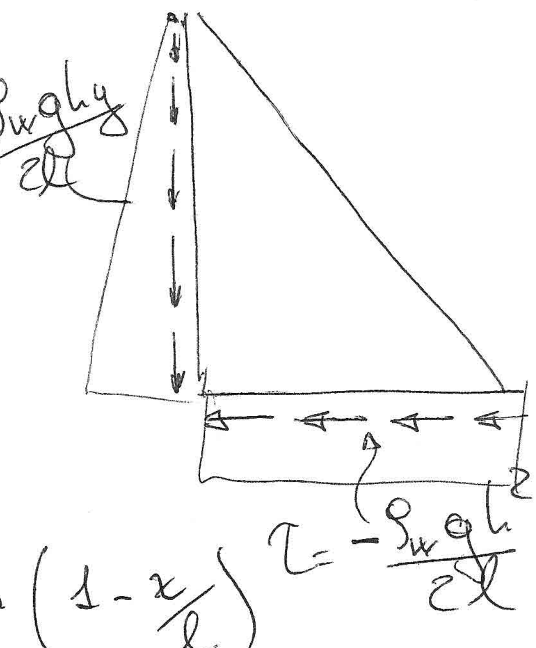
$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix} \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -\sigma_{xx} \\ -\sigma_{yx} \end{pmatrix}$$

$$\therefore \sigma_n \Big|_{AC} = -\sigma_{xx}(0,y) = -\rho_w g y$$

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix} \begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -\sigma_{xy} \\ -\sigma_{yy} \end{pmatrix}$$

$$\therefore \sigma_n \Big|_{AB} = -\sigma_{yy}(x,-h) = \rho_h g h \left(1 - \frac{x}{l}\right)$$





$$\tau \Big|_{AC} = -\sigma_{yx}(0,y) = \frac{\rho_w g h}{2l} y$$

$$\tau \Big|_{AB} = -\sigma_{xy}(x,-h) = -\frac{\rho_w g h^2}{2l}$$

3) $\underline{b} = \text{rot}(\underline{v}) \implies b_i = \varepsilon_{ijk} v_{k,j}$

$$\int_S \lambda b_i n_i \, dS \underset{\uparrow \text{ Gauss}}{=} \int_V (\lambda b_i)_{,i} \, dV =$$

$$= \int_V \lambda_{,i} b_i \, dV + \int_V \lambda \, b_{i,i} \, dV$$

$$b_{i,i} = \varepsilon_{ijk} v_{k,ji} = \varepsilon_{kij} v_{k,ij} =$$

$$= v_{1,23} - v_{1,32} + v_{2,31} - v_{2,13} + v_{3,12} -$$

$$- v_{3,21} = 0$$

$$\therefore \boxed{\int_S \lambda \, b_i n_i \, dS = \int_V \lambda_{,i} b_i \, dV \quad \text{para } b_i = \varepsilon_{ijk} v_{k,j}}$$